Rydberg constant

Laser spectroscopy and interferometry

Laser spectroscopy and optical frequency measurement

CODATA 2002 and 2006
The energy levels of hydrogen atom

\[ E_n = i \frac{R_1}{n^2} \frac{hc}{\alpha} \]

\[ E_{n;j;l} = E_{n;j} + E_n + L_{n;j} \]

Lamb shift

- QED corrections, relativistic recoil
- Nuclear size: charge radius of the proton
## Lamb shift of the 1S level

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-energy (one loop)</td>
<td>8 383 339.466 kHz</td>
<td>0.083 kHz</td>
</tr>
<tr>
<td>Vacuum polarization (one loop)</td>
<td>-214 816.607 kHz</td>
<td>0.005 kHz</td>
</tr>
<tr>
<td>Recoil corrections</td>
<td>2 401.782 kHz</td>
<td>0.010 kHz</td>
</tr>
<tr>
<td>Proton size</td>
<td>1 253.000 kHz</td>
<td>50.000 kHz</td>
</tr>
<tr>
<td>Two-loop corrections</td>
<td>731.000 kHz</td>
<td>3.300 kHz</td>
</tr>
<tr>
<td>Radiative-recoil corrections</td>
<td>-12.321 kHz</td>
<td>0.740 kHz</td>
</tr>
<tr>
<td>Vacuum polarization (muon)</td>
<td>-5.068 kHz</td>
<td>&lt;0.001 kHz</td>
</tr>
<tr>
<td>Vacuum polarization (hadron)</td>
<td>-3.401 kHz</td>
<td>0.076 kHz</td>
</tr>
<tr>
<td>Proton self-energy</td>
<td>4.618 kHz</td>
<td>0.160 kHz</td>
</tr>
<tr>
<td>Three-loop corrections</td>
<td>1.832 kHz</td>
<td>1.000 kHz</td>
</tr>
<tr>
<td>Nuclear-size corrections to SE and VP</td>
<td>-0.149 kHz</td>
<td>0.011 kHz</td>
</tr>
<tr>
<td>Proton polarization</td>
<td>-0.070 kHz</td>
<td>0.013 kHz</td>
</tr>
<tr>
<td>1S Lamb shift</td>
<td>8 172 894.560 kHz</td>
<td></td>
</tr>
</tbody>
</table>

Determination of the Rydberg constant from the 1S-2S and 2S-nD intervals (I)

The principle is to use the scaling law of the Lamb shift. The Lamb shifts vary approximately as $1/n^3$ and the deviation to this law as been calculated very precisely, and is independent of the charge distribution of the proton.


\[
\Delta_2 = L(1S) - 8L(2S) = -187.232 \text{ (5) MHz}
\]

\[
\begin{align*}
\Delta_2 = & \ h\nu(2S - 8D) \approx (1/4 - 1/64)hcR_\infty + L(nD) - L(2S) & \\
\Delta_2 = & \ h\nu(1S - 2S) \approx 3/4hcR_\infty + L(2S) - L(1S)
\end{align*}
\]

\[
7\ h\nu(2S - 8D) - h\nu(1S - 2S) = 7\times 5.9 = 41.3 \text{ kHz}
\]

\[
\begin{bmatrix}
7[E_{\text{Dirac}}(8D) - E_{\text{Dirac}}(2S) + E_R(8D) - E_R(2S)] + 7L(8D) \\
- [E_{\text{Dirac}}(2S) - E_{\text{Dirac}}(1S) + E_R(2S) - E_R(1S)] \\
\end{bmatrix} + \Delta_2 = 5 \text{ kHz}
\]

\[
\frac{\Delta R_\infty}{R_\infty} \approx 1.4 \times 10^{-11}
\]

\[
\approx \frac{57}{64} hcR_\infty
\]
Determination of the Rydberg constant from the 1S-2S and 2S-nD intervals (II)

- This method is independent of the size of the proton and few dependant from the theory.
- The method is valid for hydrogen and deuterium. From the measurements of the 1S-2S, 2S-8D, and 2S-12D transitions in hydrogen and deuterium, we obtain with a least squares adjustment:
  \[
  \frac{\Delta R_\infty}{R_\infty} \approx 10^{-11}
  \]
- The method gives also a determination of the Lamb shift. From the optical frequency measurements, we obtain:
  \[
  L(1S) = 8172.837(26) \text{ MHz}
  \]

Then it is possible to deduce a value of the charge distribution of the proton:

\[
R_p = 0.8760(78) \text{ fm}
\]

CODATA 2006
Two possibilities to surpass this limit:

- Measurement of the 2S Lamb shift in muonic hydrogen. The goal is the determination of the radius of the charge distribution of the proton $r_p$. If we measure $r_p$ with an accuracy of $10^{-3}$, we deduce $R_\infty$ with an uncertainty of $2 \times 10^{-12}$. Large collaboration at Paul Scherrer Institut.

- Measurement of the optical frequency of the 1S-3S transition. Now our accuracy is 36 kHz ($1.2 \times 10^{-11}$). Our goal is to reduce this uncertainty at 1 kHz to reduce the uncertainty in $R_\infty$ by a factor two.
2S-2P Lamb shift in muonic hydrogen $\mu$-$p$

Bohr radius: $a_0/207$

Lamb shift = self-energy + vacuum polarization + proton radius

<table>
<thead>
<tr>
<th>2S-2P</th>
<th>self-energy</th>
<th>vacuum pol.</th>
<th>$r_p$</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>e $p$</td>
<td>1085.8 MHz</td>
<td>-26.9 MHz</td>
<td>0.146 MHz</td>
<td>1057.8 MHz</td>
</tr>
<tr>
<td>$\mu$ $p$</td>
<td>0.1 THz</td>
<td>-49.94 THz</td>
<td>0.93 THz</td>
<td>-49.05 THz</td>
</tr>
</tbody>
</table>

Data acquisition in 2002, 2003 and 2007
No observed signal
Run in April – July 2009

F. Biraben, P. Indelicato, L. Julien, E.O. Le Bigot, F. Nez, C. Schwob
$\Delta \nu_{Doppler} = -\frac{V^2}{2c^2} \nu_0$

$\vec{E} = \vec{V} \times \vec{B}$

$\Delta \nu_{Stark} \propto V^2 B^2$

1S-3S experiment

Laser TiSa

LBO cavity

BBO cavity

820 nm (2W)

410 nm (1W)

205 nm (20 μW)

H(1S)

PMT

Frequency comb control by H maser Syrte

656 nm (100 ph/s)

1S 3S 2P

$\vec{B}$
Measurement of the 1S-3S frequency

Expected value: 2 922 742 936 722.6 (1.4) kHz

Measurement: 2 922 742 936 706 (5) kHz

O. Arnoult, F. Biraben, L. Julien, F. Nez
Measurement of the 1S-3S frequency

Expected value: 2 922 742 936 722.6 (1.4) kHz

Measurement: 2 922 742 936 706 (5) kHz
- New UV source at 205 nm

![Diagram showing laser interactions](image)

- Frequency doubled Nd:YVO₄ laser
- Ti:Al₂O₃ laser
- BBO
- SFG
- 205 nm

- or new crystal KBBF in the range 175 – 210 nm

- Cooling of the atomic beam