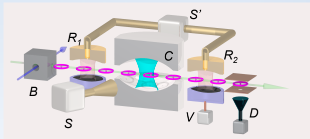
Past quantum state analysis of the photon number evolution in a cavity

I. Dotsenko, T. Rybarczyk, B. Peaudecerf, M. Penasa, S. Gerlich,
B. Julsgaard , K. Mølmer, S. Gleyzes, M. Brune, JM. Raimond, S. Haroche

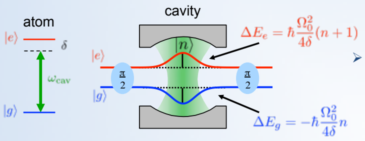
Laboratoire Kastler Brossel
Physique quantique et applications

Introduction: Quantum Non-Demolition photon-number measurement

System
High-Q superconducting microwave cavity C, circular Rydberg atoms prepared one by one in B, and Ramsey interferometer R_1-R_2-D



Dispersive interaction
Light-shift effect maps a cavity photon number n into the phase of the atomic coherence $|g\rangle$- $|e\rangle$

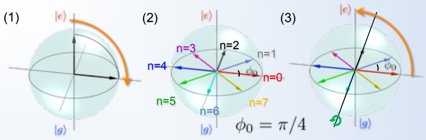


➤ Phase shift between levels $|g\rangle$ and $|e\rangle$:

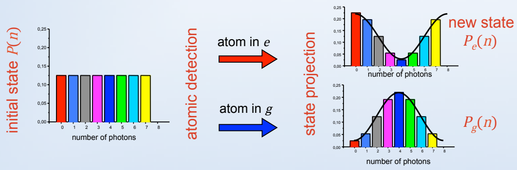
$$\phi(n) = (n + 1/2)\phi_0 \qquad \phi_0 = \frac{\Omega_0^2}{2\delta} t_{int}$$

phase shift per photon — vacuum Rabi frequency — interaction time

Ramsey interferometry
(1) 1st Ramsey zone: a $\pi/2$ pulse transfers the atom into $|e\rangle+|g\rangle$
(2) Cavity: atom undergoes phase shift $\phi(n)$
(3) 2nd Ramsey zone: $\pi/2$ pulse with a tunable phase ϕ_r followed by state-sensitive detection of the atom



One-atom measurement
Partial information on photon-number distribution $P(n)$



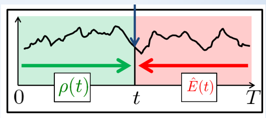
Repeating the measurement with many atoms on the same field realization progressively projects its initially broad $P(n)$ into a random photon-number state

Reference : S. Haroche and J.M. Raimond, *«Exploring the Quantum»* (Oxford Univ. Press, 2006)

Past quantum state

Idea
A quantum system can be monitored through repeated interactions with meter systems. The state of the system at time t, represented by the density matrix $\rho(t)$, then becomes conditioned on the information obtained by the meters until that time.

More insight in the state of the system at any time t is provided, however, by taking into account the full detection of all meters interacting with the system both in the past and in the future of t.



Result
Probability of result n based on the past of t

$$P^f(n,t) = \mathrm{Tr}[\hat{\Omega}_n^\dagger\hat{\Omega}_n\rho(t)]$$

Probability of result n based on both the past and the future of t

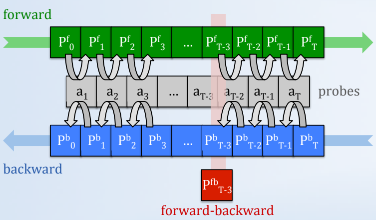
$$P^{fb}(n,t) = \frac{\mathrm{Tr}[\hat{\Omega}_n\rho(t)\hat{\Omega}_n^\dagger E(t)]}{\sum_m \mathrm{Tr}[\hat{\Omega}_m\rho(t)\hat{\Omega}_m^\dagger E(t)]}$$

Case of diagonal matrices in the measurement basis (*e.g.* photon-number distribution and QND measurement)

$$P^{fb}(n,t) = \frac{P^f(n,t)\,P^b(n,t)}{\sum_m P^f(m,t)P^b(m,t)}$$

Reference : S. Gammelmark *et al.*, PRL **111**, 160401 (2013)

Forward-backward analysis of photon number distribution



Probability of a photon-number measurement
product of two distributions:

$$P^{fb}(n,t) \propto P^f(n,t)P^b(n,t)$$

Uniform initial distribution
no *a priori* knowledge on the field:

$$P^f(n,0) = P^b(n,T) = 1/N$$

Two processes taken into account at each detection

Measurement at time t_i (*Bayes' rule*)

$$P^f(n,t_i^+) \propto P(a_i|\phi_{r_i},n)P^f(n,t_i^-)$$
$$P^b(n,t_j^-) \propto P(a_j|\phi_{r_j},n)P^b(n,t_j^+)$$

with

$$P(g|\phi_r,n) = \cos^2[\varphi_0(n+1/2) - \phi_r]$$

$a_i \in \{g,e\}$ - detection result of atom i
ϕ_r - Ramsey phase used for this atom

Decoherence

$$\frac{dP^f(n,t)}{dt} = \sum_m K_{n,m}P^f(m,t)$$
$$\frac{dP^b(n,t)}{d(-t)} = \sum_m K_{m,n}P^b(m,j)$$

with relaxation matrix:

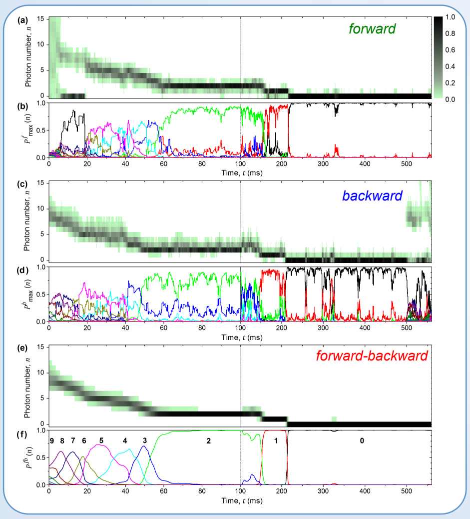
$$K_{n,n} = -\kappa[(1+n_b)n + n_b(n+1)]$$
$$K_{n,n+1} = \kappa(1+n_b)(n+1)$$
$$K_{n,n-1} = \kappa n_b n$$

$\kappa = 1/T_c$ cavity relaxation rate with T_c=65 ms
n_b= 0.06 thermal photon number

Improved resolution of the photon-number evolution

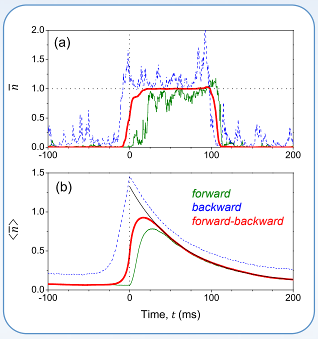
Estimation of photon number distribution

Initial field (coherent state with 7 photons on average) decays to vacuum state



Resolution of quantum jumps

Initial field at t = -400ms is vacuum
At t = 0 a resonant atom in state e is sent to inject a photon



1 single trajectory

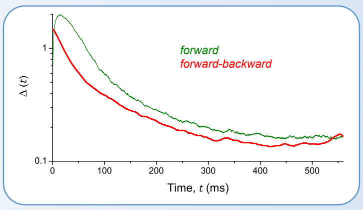
PQS: nearly prefect resolution of the jump time

average over 3000 trajectories

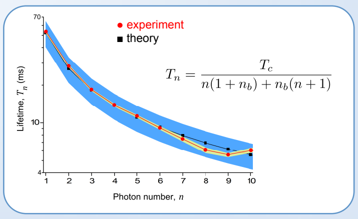
Results
- significant noise reduction
- higher purity of the reconstructed state
- lifted ambiguities due to the periodicity of the interferometric measurement
- access to larger photon numbers
- highly improved resolution of the quantum jumps

Additional validation

Improved correlation between distributions reconstructed from two independent subsets of data measured on the same experimental realization



Photon-number lifetimes extracted from the statistics of individual resolved quantum jumps



References : T. Rybarczyk *et al.*, PRA 91, 062116 (2015))

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