Interrogation Oscillator Noise Rejection in the Comparison of Atomic Fountains

S. Bize, Y. Sortais, P. Lemonde, S. Zhang, Ph. Laurent, G. Santarelli, C. Salomon*, A. Clairon.

BNM-LPTF, Observatoire de Paris, 61, Av. de l'Observatoire, 75014 Paris, France.
*LKB-ENS, 24, Rue Lhomond, 75231 Paris, France.

ABSTRACT

The frequency stability of an atomic fountain clock can be limited by the phase noise of the interrogation oscillator via the "Dick Effect". In this paper we demonstrate the rejection of the phase fluctuations of the interrogation oscillator by the synchronisation of atomic fountains. A reduction by a factor of 16 in the Allan standard deviation of the relative frequency difference between two fountains has been obtained.

1. INTRODUCTION

The development of new passive frequency standards using trapped ions or cold atoms has produced devices with a potential fractional frequency stability in the range of $10^{-14}\tau^{-1/2}$ [1-5]. In these new types of standards, the atomic interrogation process and the control of the interrogation oscillator are periodic, with period T_c.

In the late eighties, J. G. Dick [6,7] showed that the oscillator frequency noise at Fourier frequencies which are close to multiples of $1/T_c$ is down-converted, leading to a degradation of the frequency stability.

This work motivated new thoughts on this subject and several recent papers have reported a more complete description of this phenomenon, called the "Dick effect" [8-10].

If limited by interrogation noise, the Allan variance of the atomic frequency standard is related to the frequency noise of the interrogation oscillator in the following way:

$$\sigma^2_{y\,\mathrm{lim}}(\tau) = \frac{1}{\tau}\sum_{m=1}^{\infty}\left(\frac{{g^c_m}^2}{g_0^2} + \frac{{g^s_m}^2}{g_0^2}\right) S_y^{IO}(m/T_c) \qquad (1)$$

where $S_y^{IO}(m/T_c)$ is the one-sided power spectral density of the relative frequency fluctuations of the interrogation oscillator at Fourier frequencies m/T_c, and the parameters g_0, g^s_m and g^c_m are defined by:

$$\begin{pmatrix} g^s_m \\ g^c_m \end{pmatrix} = \frac{1}{T_c}\int_0^{T_c} g(\xi)\begin{pmatrix} \sin 2\pi m\,\xi/T_c \\ \cos 2\pi m\,\xi/T_c \end{pmatrix} d\xi \,,$$

$$g_0 = \frac{1}{T_c}\int_0^{T_c} g(\xi)\,d\xi \,. \qquad (2)$$

Here $g(\xi)$ is the sensitivity function. It links the frequency fluctuations of the interrogating field $\delta\omega(t)$ to the fluctuations of the atomic transition probability δP:

$$\delta P = \frac{1}{2}\int_{\mathrm{int.}} g(t)\,\delta\omega(t)\,dt \qquad (3)$$

The feedback loop converts these probability fluctuations into noise in the frequency standard.

With the state-of-the-art BVA quartz oscillators, the best stability which has been obtained with an atomic fountain is about $1.1\ 10^{-13}\tau^{-1/2}$.

To overcome this limitation a lower noise interrogation oscillator can be used. For example with a cryogenic sapphire oscillator, the BNM-LPTF atomic fountain FO1 achieved a frequency stability of $4.5\ 10^{-14}\tau^{-1/2}$ [5].

Alternatively, one can synchronise the operation cycle of a pair of identical frequency standards, interrogating the atoms at the same time with the same oscillator. The frequency fluctuations due to the interrogation oscillator are then correlated and the comparison between the two standards can be made free of this effect. This idea was independently proposed by L. Maleki and co-workers [8].

In this paper we present the first experimental realisation of this scheme. Two atomic fountains have been synchronised to reject the noise of the common interrogation oscillator. Three independent fountain at the BNM-LPTF have been compared in this way: the primary Cs standard FO1, a Rb fountain [9] and a transportable Cs fountain [10].

2. EXPERIMENTAL SET-UP

To implement the rejection technique (Fig. 1), we distribute a single interrogation oscillator signal to all of the atomic fountains with a 100 MHz low noise link of 200 m length. A synchronisation pulse is generated at the beginning of each cycle by FO1 which triggers the cycle sequence of the other fountains. The stability of the synchronisation is better than one ms.

In order to demonstrate the noise rejection, we use a noisy quartz oscillator with an Allan standard deviation of about $5\ 10^{-12}$ at 1 second. The "Dick Effect" results in an Allan deviation of $2.4\ 10^{-12}\tau^{-1/2}$, almost two orders of magnitude higher than other limitations to the frequency stability of each fountain.

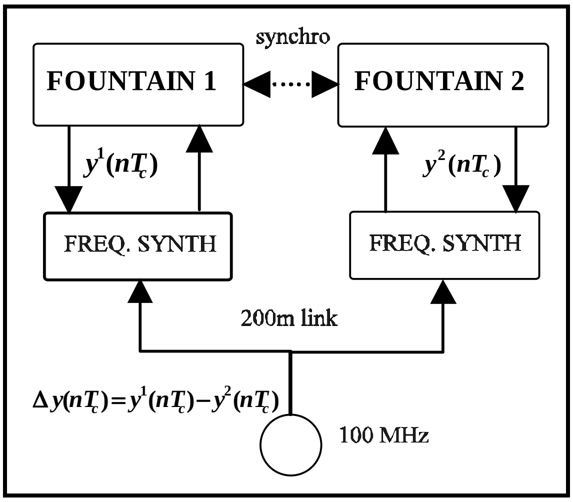

Figure 1: Schematic of the link connecting the interrogation oscillator to the two fountains.

3. RESULTS

Figure 3 shows the quadratic sum of the Allan deviation of the Rb and FO1 fountains compared against a H-maser that would result from the unsynchronised comparison. Also plotted is the Allan deviation of the relative frequency difference between the fountains. We observe an improvement of the frequency comparison by a factor of 16. The stability in the comparison is $2\ 10^{-13}\tau^{-1/2}$ close to the value obtained with the best quartz oscillators. In the comparison between the Rb fountain and the transportable Cs fountain, a preliminary factor of 10 improvement was obtained.

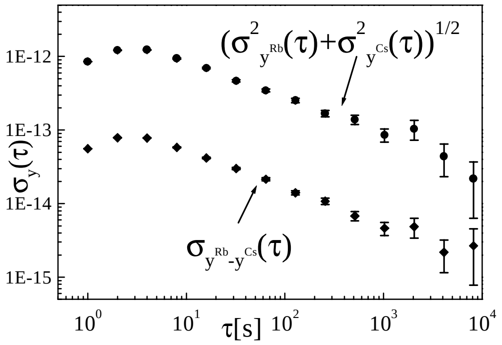

Figure 3: Dots: quadratic sum of the Allan deviation of the Rb and Cs fountains. Diamonds: Allan deviation of the frequency difference between the fountains.

When a delay between the two interrogation cycles is applied, the sensitivity functions of the two fountains are not perfectly matched (Fig. 2). The relative frequency fluctuations of both standards are then partially decorrelated. Figure 4 shows the noise rejection plotted against the delay between the operation cycles of the two fountains. The rejection is defined here as the quadratic sum of the Allan deviation of each fountain divided by the Allan deviation of the frequency difference.

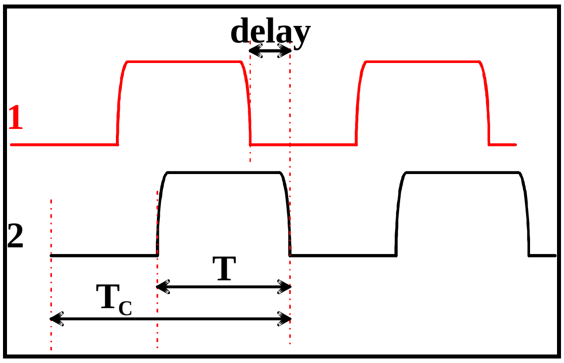

Figure 2: Sensitivity functions of two fountains.

The continuous line is a numerical simulation of the rejection using the measured phase noise spectral density of the interrogation oscillator and a simple model of the sensitivity functions of the two fountains. This model assumes that the atoms are subjected to square microwave pulses of length equal to the actual time spent in the interrogation cavity. We observe a good qualitative agreement between experiment and theory. A more refined model requires a very precise knowledge of the oscillator noise spectrum and of the actual sensitivity functions, the latter can be precisely measured using previously developed techniques [12]. This graph also shows that since the optimum operation point is quite sharp, a few millisecond synchronisation is needed for a significant rejection of the interrogation oscillator noise. This indicates that high order harmonics of the noise are involved in the rejection process.

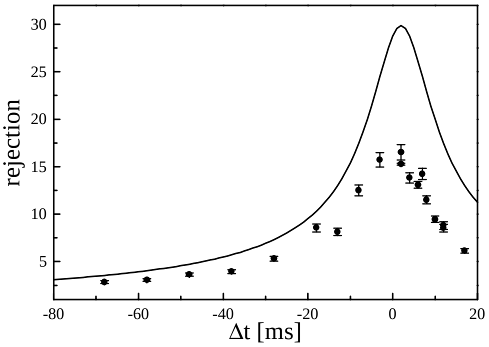

Figure 4: Rejection versus the delay between the operation cycles (dots). The solid line is a numerical simulation using the noise measurements of the interrogation oscillator.

Finally, it is worth noting that the rejection of the interrogation oscillator does not result in a more stable clock; the output of each individual fountain is still affected by the interrogation oscillator frequency noise.

4. CONCLUSIONS

We have presented preliminary results of interrogation oscillator noise rejection by the synchronisation of two atomic fountains. By performing the synchronisation with a 10^{-13} stability quartz oscillator, it seems possible to reach the quantum projection noise limit in the comparison between the fountains. With three synchronised fountains, the frequency stability of each individual clock could be determined with a three corner hat measurement. This opens the way to a frequency stability of a few $10^{-14}\,\tau^{-1/2}$ in the comparison between fountains, a prerequisite for an accuracy evaluation approaching 10^{-16}.

5. ACKNOWLEDGEMENTS

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