ATOMIC PARITY VIOLATION
PAST, PRESENT STATUS AND FUTURE PERSPECTIVES

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In this paper, we review the progress made in the determination of the weak charge, Q_W, of the cesium nucleus which raises the status of Atomic Parity Violation measurements to that of a precision electroweak test. Not only is it necessary to have a precision measurement of the electroweak asymmetry in the highly forbidden 6S-7S transition, but one also needs a precise calibration procedure. This requires in practice to compare the parity violating amplitude E_1^{PV} to M_1^{hf}, a parity conserving amplitude induced by hyperfine interaction, whose absolute value is precisely known on theoretical grounds. In addition, atomic calculations necessary for extracting Q_W from E_1^{pv} have to be precisely tested in order to estimate their uncertainty. The 0.6% accurate determination of Q_W, announced in 1999 by the Boulder group, marks an important progress in APV measurements, allowing the exploration of new areas of electroweak physics. It has prompted atomic physicists to reexamine the calculations, as summarized here. Current perspectives in the experimental field are presented.

1 Introduction

We begin with a rapid summary of the main questions addressed in this introductory paper. First, we indicate the connection between Atomic Parity Violation (APV) and Electroweak Theory. Everything centres around one electroweak parameter, the weak charge of an atomic nucleus, Q_W. Then we provide a short background to APV experiments. How can Z_0 bosons affect radiative atomic transitions? What can be deduced from a precise determination of Q_W? Next, how is Q_W extracted from experiment? Besides the measurement of a left-right asymmetry itself, A_{LR}, a calibration procedure is absolutely necessary. In addition, the theoretical interpretation of the result requires state of the art atomic physics calculations. Then, we shall present the most precise result obtained recently in cesium by the Boulder group [1,2] and discuss its implications for electroweak theory. As will be seen, the deviation between this result and the Standard Model prediction triggered important theoretical efforts. Finally, we mention other projects under current development worldwide, among them another Cs experiment in Paris, which has recently reached the stage where it provides PV data by a method totally different from Boulder. There are also proposals for doing measurements in francium and other radioactive alkali isotopes by using the techniques of laser cooling and trapping.

2 Electroweak parity violation in atoms

For a naive estimate of A_{LR} let us consider two hadronic radiative transitions, the first one, of amplitude A_{em}, governed exclusively by electromagnetic processes, and the second one, of amplitude A_W, associated with a Z_0 boson exchange (see Figure 1). The weak amplitude contains

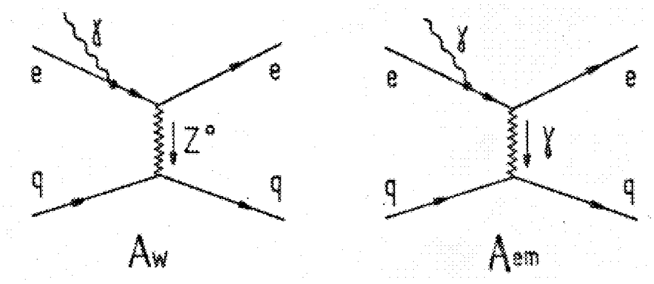

Figure 1: Schematic representation of the two amplitudes which can contribute to the same radiative process and give rise to an electroweak interference.

a part which is odd under space reflexion, which will give rise to a left-right asymmetry A_{LR} by interference with the dominant electromagnetic amplitude. For two mirror-image experiments we obtain two different transition probabilities: $P_{L/R} = |A_{em} \pm A_W^{odd}|^2$ and we find:

$$A_{LR} = \frac{P_L - P_R}{P_L + P_R} = 2\,Re\left(A_W^{odd}/A_{em}\right) \; . \tag{1}$$

If q denotes the four-momentum transfer between the lepton and the hadron, A_{em} is proportional to e^2/q^2 while $A_W \propto g^2/\left(q^2 + M_{Z_0}^2 c^2\right)$ with $g^2 \approx e^2$, hence $A_{LR} \simeq q^2/M_{Z_0}^2 c^2$. In atoms we expect q to be given by the inverse of the Bohr radius, i.e. $q \sim m_e \alpha c$. For the left-right asymmetry we thus arrive at an exceedingly small value :

$$A_{LR} \simeq \alpha^2 \frac{m_e^2}{M_{Z_0}^2} \approx 10^{-15} \; .$$

Such a result would appear to make the observation of the left-right asymmetry in atoms completely hopeless. Fortunately there are important enhancement mechanisms which make this naive estimate far too pessimistic. In fact, in actual experiments, A_{LR} can be as large as a few times 10^{-6}.

The first source of enhancement finds its origin in the so-called Z^3 law, that we predicted Claude Bouchiat and I in 1974 [3,4]. It states that the electroweak effects in atoms should grow a little faster than the cube of the atomic number Z. Indeed, for valence electrons belonging to penetrating orbitals, like $s_{1/2}$ or $p_{1/2}$, the orbital is deformed in the vicinity of the atomic nucleus, right where the short range interaction takes place. It looks like the orbital associated with a Coulomb potential of charge Z, whose radius is given by a_0/Z. Hence, in heavy atoms the factor $|q^2|$ is enhanced by the factor Z^2. Moreover, the various nucleons in the atomic nucleus add their contributions coherently. Since, in heavy atoms, the number of nucleons grows roughly as Z, the overall enhancement effect is proportional to Z^3.

For a more quantitative analysis it is necessary to introduce the parity violating electron-nucleus potential which, in the the non-relativistic limit, can be written as:

$$V_{pv}(r_e) = \frac{Q_W\, G_F}{4\sqrt{2}} \left(\delta^3(\vec{r_e})\, \vec{\sigma}_e . \vec{v}_e / c \, + \, h.c. \right) \; . \tag{2}$$

Here we have kept the dominant contribution in which the Z_0 couples to the electron as an axial vector and to the nucleons as a vector. Consequently, the strength of this interaction is naturally expressed in terms of a nuclear charge, Q_W. For the electroweak interaction this charge plays the same role as the nuclear electric charge for the Coulomb interaction. Hence its name, the weak nuclear charge. Like the electric nuclear charge, Z, the weak charge Q_W is the sum of the weak charges of all the constituents of the atomic nucleus, the u and d quarks :

$$Q_W = (2Z + N) Q_W(u) + (Z + 2N) Q_W(d) \; . \tag{3}$$

In the Standard Model it so happens that Q_W lies close to the neutron number[a] :

$$Q_W(SM) = -N - Z\left(4\sin^2\theta_W - 1\right) \simeq -N\ . \tag{4}$$

The second source of enhancement comes from the possibility of exciting highly forbidden transitions like the $6S_{1/2} \to 7S_{1/2}$ transition in cesium. In a transition such as $6S_{1/2} \to 7S_{1/2}$ the electromagnetic selection rules strictly forbid the existence of an electric dipole transition. The weak interaction associated with Z_0 exchange breaks this rule and gives rise to a parity violating electric dipole amplitude[b], E_1^{pv}. This E_1 amplitude is of course very small, 10^{-11} in atomic units, ea_0. On the other hand, symmetry allows the existence of a magnetic dipole amplitude, M_1. However, because the two states connected by the transition have different radial quantum numbers, the M_1 transition is suppressed: its amplitude is only $4 \times 10^{-4}\ \mu_B/c$, so that we can anticipate a relatively large asymmetry: $Im\ E_1^{pv}/M_1 \simeq 0.5 \times 10^{-4}$. Even today, cesium appears to be a good compromise between a high atomic number necessary to have sizable effects and the simple atomic structure required to make precise atomic calculations.

Once the Z^3 enhancement became apparent, the main question was how best to take advantage of it. There were in fact two different lines of attack. The first takes advantage of highly forbidden M_1 transitions such as $6S_{1/2} \to 7S_{1/2}$ in cesium. Its merits are the relatively large asymmetry and the simple atomic structure characteristic of an alkali atom which has a single valence electron outside a tight atomic core. This type of transition, however, represented completely new territory and the suppression factor looked absolutely huge, $\sim 10^{14}$, so that one could anticipate difficulties with the signal-to-noise ratio. Nevertheless, this was the approach chosen by our group in Paris and later on by the Boulder group[5]. The forbidden M_1 $6P_{1/2} - 7P_{1/2}$ transition in Tl was selected by the Berkeley group[6]. The second line of attack consists in working with allowed M_1 transitions in atoms of even higher Z such as Tl, Pb and Bi, with Z respectively equal to 81, 82, and 83. The suppression factor is only 10^5 and *a priori* this should avoid the signal-to-noise difficulties. This approach was adopted at Oxford[8], Seattle[9], Novosibirsk[10] and Moscow[11]. Precise measurements in these elements have been achieved, but presently the difficulty lies in the more complicated atomic structure. The accuracy in Q_W is today limited by the precision in atomic calculations. For this reason this presentation is limited to the case of cesium where the most precise determination has been so far achieved. Hereafter, we shall concentrate on the $6S_{1/2} \to 7S_{1/2}$ transition in cesium, for which $E_1^{pv} \simeq 10^{-11} i\ ea_0$ and $M_1 \simeq 2 \times 10^4\ E_1^{pv}$.

3 APV in the highly forbidden cesium transition

In order to suppress a background coming from loosely bound cesium dimers, we apply a static electric field $\mathbf{E}$. This field induces a transition electric dipole $\mathbf{d}^{ind}$ via parity-conserving mixing of atomic P states with S states. In this induced dipole

$$\mathbf{d}^{ind} = \alpha\mathbf{E} + i\ \beta\ \vec{\sigma} \times \mathbf{E}\ , \tag{5}$$

there are actually two contributions, one due to the scalar transition polarizability α, and the other to the vector polarizability β. Here $\vec{\sigma}$ stands for the electron spin operator. The coupling of $\mathbf{d}^{ind}$ to the electric radiation field $\vec{\epsilon}$ provided by a resonant laser wave, gives rise to an induced electric dipole amplitude, $E_1^{ind} = \mathbf{d}^{ind} \cdot \vec{\epsilon}$. An excellent control of E_1^{ind} can be achieved by adjusting the strength of $\mathbf{E}$, its direction with respect to both the direction of the light beam

[a] This formula is valid only to lowest order in the electroweak interaction.

[b] Note that Z_0 exchange gives rise to a transition electric dipole, not to a static electric dipole. The reason is that this PV weak interaction preserves time reversal invariance, while a static dipole would be a manifestation of simultaneous P and T violation. Another consequence of time reversal invariance is that E_1^{pv} is pure imaginary in a phase convention where M_1 is real.

and the beam polarization $\vec{\epsilon}$. Reversing the direction of **E** can even give a specific signature to all effects linear in E_1^{ind}. For all these reasons $E_1^{ind}E_1^{pv}$ has become the kind of interference effect detected in all cesium APV experiments so far. The transition rate $|E_1^{ind}|^2$ grows like the square of the electric field, while the asymmetry E_1^{pv}/E_1^{ind} is inversely proportional to the field, but still can reach values well above 10^{-6}.

The experiment completed in Paris [12], in 1982 and 1983, was the the first carried out in cesium. With 12% experimental accuracy and a theoretical uncertainty at that time less than 8 %, it has led to a quantitative test of the Standard electroweak theory in the electron-hadron sector, which is of a new kind[3]. First, because of the low momentum transfers involved ($\simeq$ 1 MeV/c) it extends considerably the range of $|q^2|$ where the theory finds experimental support. Moreover, information deduced from Q_W complements that obtained from high energy experiments because, as mentioned above (Eq. 3), in atoms all the nucleons and all the quarks act coherently [13]. Thus it is clear that *atomic physics provide a unique test of electroweak theory in the electron-hadron sector and in the long range limit.* In particular, it provides a test for additional neutral vector bosons. Most extensions of the Standard Model predict the existence of additional bosons. There are actually several types of models. In those where $M_{Z'}$ is arbitrarily larger than M_Z and $g_{Z'} \approx g_Z$, Q_W receives a contribution from the Z' which is inversely proportional to the square of its mass, $\Delta Q_W = Q_W M_Z^2/M_{Z'}^2$. Therefore APV can give a lower limit to $M_{Z'}$, even without any $Z - Z'$ mixing [14,15,16]. Other models predict the existence of light or even very light gauge bosons [17]. In the past these particles were invoked as a possible source of the fifth force, for $\hbar/M_U c \geq 0.1$ m (*i.e.* $M_U \approx 10^{-7}$ eV). In this context APV can be seen as a probe of an intermediate mass range [13]: $M_U \geq 0.1$ MeV.

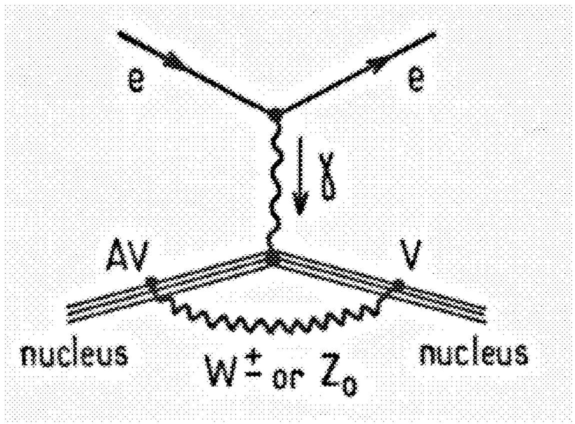

Figure 2: Schematic representation of the nuclear-spin-dependent PV interaction associated with the nuclear anapole moment.

APV can also be a source of valuable information relevant for nuclear physics. This aspect concerns the nuclear-spin-dependent PV interaction. The relevant experimental parameter involved here, is the difference between the two asymmetries measured on two different hyperfine lines belonging to the same transition [c]. If non-zero the quantity:

$$r_{hf} = \frac{A_{LR}(6S \to 7S, \Delta F = -1)}{A_{LR}(6S \to 7S, \Delta F = +1)} - 1 \ , \qquad (6)$$

is a manifestation of the nuclear-spin-dependent interaction. An effect of about 4% is expected as a result of the parity violating interactions taking place inside the nucleus between the quarks. The atom is contaminated by photon exchange, (see the corresponding diagram represented on Fig. 2). The theoretical concept relevant for describing this effect is the nuclear anapole

[c] The cesium natural isotope having the nuclear spin 7/2, the $6S_{1/2}$ and $7S_{1/2}$ states posses two hyperfine substates with total angular momentum F =3 and 4.

Table 1: Results of atomic calculations in Cesium: $\mathcal{E}_1^{pv} = (-N/Q_W)\, Im\, E_1^{pv}\ (10^{-11}|e|a_0)$

Semi - Empirical	First Principles
$-0.935 \pm 0.02 \pm 0.03$ [(a)] Paris 86	-0.908 ± 0.010 [(d)] Novosibirsk 89
-0.904 ± 0.02 [(b)] Oxford 90	-0.905 ± 0.009 [(e)] Notre Dame 90
-0.895 ± 0.02 [(c)] Paris 91	

(a) Bouchiat and Piketty [24]; (b) Hartley and Sandars [25]; (c) Bouchiat (1991); (d) Dzuba *et al.* [26]; (e) Blundell *et al.* [27].

moment, introduced a long time ago by Zel'dovich [18]. For a simple interpretation in terms of chiral magnetization of the nucleus the reader is referred to [19]. Explicit calculations have been performed for cesium by the Novosibirsk [20,21] and Paris [22,23] groups. It is interesting to note that the effect of the electron-nucleus Z_0 exchange associated with an axial-vector coupling to the nucleons, also involving the nuclear spin, is formally identical but it is about five times smaller.

4 Status of atomic physics calculations in the early 1990's

Atomic physics calculations are essential to extract Q_W from the experiment. The quantity E_1^{pv} can be considered as an infinite sum over the intermediate P states admixed with the S states by the parity-violating interaction :

$$E_1^{pv} = \sum_n \frac{\left\langle 7S_{1/2}\right| d_z \left|nP_{1/2}\right\rangle \left\langle nP_{1/2}\right| V_{pv} \left|6S_{1/2}\right\rangle}{E\left(6S_{1/2}\right) - E\left(nP_{1/2}\right)} + \text{ crossed terms .}$$

The atomic orbitals and the valence-state energies are perturbed by many-body effects. Initially, two different approaches have been employed, one semi-empirical and the second starting from first-principles. Results agree within the stated precision.

The semi-empirical approach incorporates in a consistent way different empirical data: energies of the valence states, allowed dipole matrix elements, and hfs splittings of the $nS_{1/2}$ and $n'P_{1/2}$ states. Many body perturbation theory is needed only to compute small correcting terms. The first principle approach performs relativistic many-body computations using techniques inspired from field theory. Two groups have independently achieved a real *tour de force* by reaching a precision better than 1%, for this 55 electron system (!). The results are summarized in Table 1. (For a presentation of the most recent calculations see §7).

We must stress the importance of a precise empirical determination of the allowed E_1 amplitudes and the hf splittings of the $nP_{1/2}$ states, used either as an input in the semi-empirical approach or as *test parameters in the First Principle method.* The latter, indeed, estimates its precision essentially from the deviation between experimental and calculated values of test parameters, using of course the same calculation technique as for E_1^{pv}. At the time the calculations were published, some of the data were incorrect, in particular the $7S_{1/2}$ Stark shift, determined by the Boulder group. Since then, these measurements have been improved, the erroneous data have been corrected [28] and the precision increased.

5 A precision test of electroweak theory

In the early 1990's the goal became to perform a precision test of electroweak theory from APV measurements in Cs. For this several conditions, had to be satisfied:

i) Experiments only measure amplitude ratios like $Im\ E_1^{pv}/\beta E$ and since β is known from atomic theory within 1% accuracy only, there was a crucial need for an accurately known

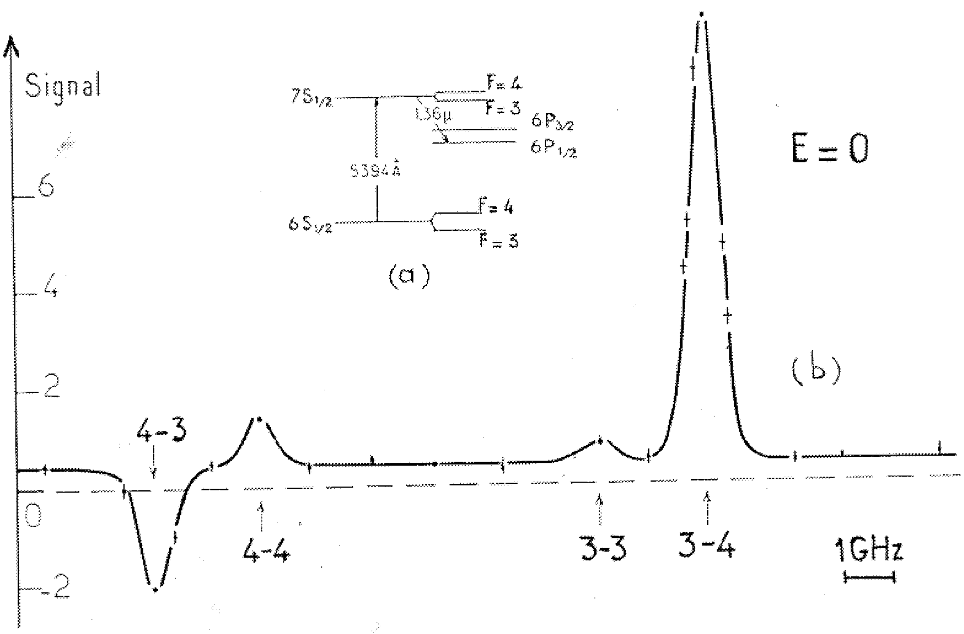

Figure 3: Hyperfine structure of the highly forbidden $6S_{1/2} \to 7S_{1/2}$ transition observed without electric field. The incident beam at 539.4 nm is circularly polarized. The plotted signal detected via the fluorescence at 1.36 μm, is a modulation proportional to the electronic polarization of the $7S$ state. The $3 \to 4$ and the $4 \to 3$ peaks involve respectively $|M_1 + M_1^{hf}|^2$ and $|M_1 - M_1^{hf}|^2$ weighted by known nuclear spin dependent factors. (Figure adapted from Bouchiat *et al.* [31]). Inset: Energy levels of cesium involved in the parity violation experiments.

amplitude usable for *absolute calibration.*

ii) There was also a need for more precise PV measurements on two hfs lines: a well-defined linear combination of the asymmetries leads to the nuclear-spin-independent contribution, and simultaneously, from the linearly independent combination, a test concerning the nuclear anapole moment is performed.

iii) There was also a need for better tests of atomic theory, so that at the light of new results the accuracy of the First Principle calculations could be reconsidered. At that time more precise calculations were looking difficult and actually not envisaged. Fortunately, as shown in 1988 by the Paris-ENS group [29,30], a very precisely known transition amplitude does exist. As already mentioned, in zero electric field the $6S_{1/2} \to 7S_{1/2}$ Cs transition is of an M_1 nature. It so happens that the M_1 amplitude receives a certain contribution $M_1^{hf} \simeq 0.19 \times M_1$, induced by the off-diagonal hyperfine interaction. Experimentally this contribution is easily identified because it contributes only to the $|\Delta F| = \pm 1$ hyperfine lines and with opposite signs [7]. The important point is that M_1^{hf} can be precisely expressed in terms of the geometrical mean of the diagonal matrix elements [29], *i.e.* the hyperfine splittings ΔW of the two S states, themselves measured in cesium with a high precision:

$$M1^{hf} = C\sqrt{\Delta W_{6S} \cdot \Delta W_{7S}}. \tag{7}$$

Corrections to this factorization rule coming from many-body effects have been evaluated [29] and found to be very small: $(0.25 \pm 0.25) \times 10^{-2}$. Two recent First Principle calculations [32,33] confirm this result, the latter with a precision of a few 10^{-4}.

Finally, as advocated in ref[30], a precise measurement of M_1^{hf}/β provides an *absolute* determination of β and makes possible an absolute determination of E_1^{pv} with a theoretical uncertainty less than 2.5×10^{-3}. While doing this kind of measurement, however, one must be very careful since there exists also an E_2 transition amplitude which is induced by the off-diagonal hyperfine interaction admixing S states with D states. Although small ($E_2/M_1^{hf} \simeq 5 \times 10^{-2}$), this amplitude must be taken into account in the data analysis [30], [d].

Figure 3 shows that, using experimental tricks, the $6S_{1/2} \rightarrow 7S_{1/2}$ Cs transition can be detected well above noise and background even in a zero electric field [31] and provides a clear manifestation of M_1^{hf}.

We now come to the precision measurements of E_1^{pv}/β made by the Boulder group [2] in 1997, on two different hyperfine lines $\Delta F = +1$ and -1. The physical parameters which define this experiment are the dc electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$, applied perpendicularly to the resonant light beam and the angular momentum, $\xi\mathbf{k}$, of this beam. The pseudo-scalar quantity which manifests parity violation in this experiment is the mixed product: $\mathbf{E} \times \mathbf{B} \cdot \xi\mathbf{k}$. It is expected to appear in the transition rate, provided that the Zeeman components of the transition are resolved, otherwise compensations occur. The experiment is performed with an atomic beam which passes through three regions (see Fig 4). First, the atoms are prepared, *i.e.* optically pumped into a single Zeeman sublevel. Next, they enter the interaction region, where they cross at right angles a laser beam at 539.4 nm, resonant for the 6S-7S transition. The laser intensity is greatly amplified by means of a Fabry-Perot build-up cavity (PBC). In the third region, one monitors the modification of population induced in the interaction region, including the transition rate and a background. Extraction of the electroweak interference term is made by reversing the signs of $\mathbf{E}$, $\mathbf{B}$ and ξ. The PBC with a finesse of 10^5 is a key component of the set-up: the transition rate is enhanced and the E_1M_1 interference which is a potential source of systematics, is suppressed. However, such a large power stored in a cavity generates side-effects. Due to ac Stark shifts the lines become asymmetric and adjacent Zeeman components overlap slightly, so delicate corrections have to be made. By the authors'own admission one of the systematic effects is not yet completely understood [2].

Combining the results quoted by the Boulder group for the two hyperfine lines $6S, F = 4 \rightarrow 7S, F = 3$ and $6S, F = 3 \rightarrow 7S, F = 4$, one can extract the nuclear-spin-independent contribution:

$$Im\ E_1^{pv}/\beta = (1.5576 \pm .0056)\mathrm{mV/cm},$$

discussed later on, as well as the r_{hf} parameter (Eq. 5):

$$r_{hf} = (4.8 \pm 0.7) \times 10^{-2} .$$

This last result provides a clear manifestation of the nuclear anapole moment, in semi-quantitative agreement with nuclear physics theoretical estimates.

Nevertheless, in view of the importance of the results concerning Particle Physics via Q_W and Nuclear PV forces via the nuclear anapole moment, and also in view of the difficulty of this type of experiment, an independent measurement appears necessary.

In addition, a very precise measurement of $M_1^{hf}/\beta E$ has been performed recently [1] by the Boulder group. The announced accuracy reaches 0.12 % in spite of very difficult experimental conditions: the presence of a background exceeding by a factor of 100 the M_1^2 transition rate to be measured, photoionization effects requiring intensity-dependent corrections and corrections necessary to account for effects connected with the E_2 amplitude. The result agrees, within error bars, with the previous 1.3% accurate, Paris determination [30].

6 Result and implications

We can now summarize the four successive steps followed by the Boulder group to arrive at the determination of $Q_W(Cs)$.

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[d] In a given experimental situation the amplitude E_2 exactly mimics M_1^{hf} so that the measured quantity is actually $M_1^{hf}(1+k\ E_2/M_1^{hf})$, where k is an angular momentum dependent computable factor which varies from one experimental configuration to another. Therefore a global fit of at least two experiments is needed to extract E_2/M_1^{hf} and to obtain the M_1^{hf} contribution [30].

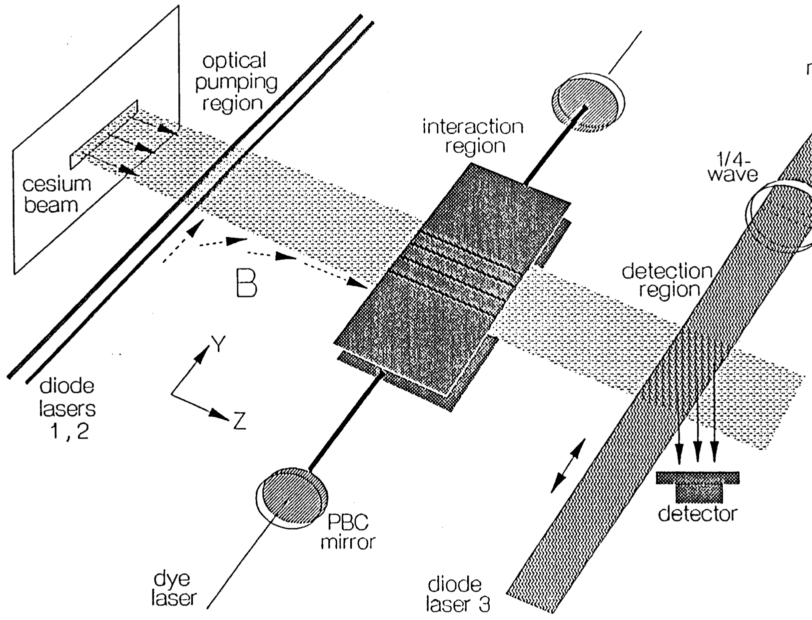

Figure 4: Latest version of the Boulder experiment in cesium, using a polarized atomic beam, Figure adapted from Wood *et al.* [2].

i) measurement of $Im\ E_1^{pv}/\beta E$ with quoted[2] uncertainty $\sigma_1 = 0.35\%$.

ii) measurement of $M_1^{hf}/\beta E$ with quoted[1] uncertainty of $\sigma_2 = 0.12\%$,

iii) use of the semi-empirical theoretical value, $M_1^{hf}(s.e.)$ from ref [29,30], known with an uncertainty $\sigma_3 = 0.25\%$

Hence, using the relation:

$$E_1^{pv}(exp) = M_1^{hf}(s.e.) \times \frac{(E_1^{pv}/\beta E)^{exp}}{(M_1^{hf}/\beta E)^{exp}},$$

one gets an absolute determination of $E_1^{pv}(exp)$.

iv) extraction of Q_W from $E_1^{pv}(exp)$, using atomic theory is the last step:

$$Q_W^{exp} = -N \times \frac{E_1^{pv}(exp)}{\mathcal{E}_1^{pv}(at.th.)}.$$

Until recently, this step was considered to be the major source of uncertainty, $\simeq 1\%$ (see Table 1). Although no new theoretical result was available, the Wieman group decided to reestimate the uncertainty of the previous calculations. By taking into account some new experimental tests, they noted that the agreement of all test parameters with theory had become very good (from 1 to 2% the largest deviations have been lowered down to 0.4% or even less). So they concluded that the theory was more accurate than previously thought, and they assigned to their result a theoretical uncertainty equal to $\sigma_4 = 0.4\%$. Nevertheless one may note that the above tests are not sufficient: as we shall see (§7) there are several effects which affect differently E_1^{pv} and the test parameters.

The result finally reported by the Boulder group is :

$$Q_W^{exp} = -72.06 \pm 0.28(exp) \pm 0.34(th).$$

When the errors of different origins are added quadratically the final uncertainty is $\sigma = 0.44$, which corresponds to a fractional accuracy of 0.6%.

This result has to be compared to the prediction of electroweak theory first obtained by Marciano and Sirlin[34]. Their evaluation has been recently updated[36,37] to incorporate the value of the top quark mass from the Standard Model fit of electroweak data and an estimate of the Higgs boson mass ($m_t = 172.9 \pm 4.6$ GeV, $M_H = 98^{+57}_{-38}$ GeV). The result is:

$$Q_W^{th} = -73.09 \pm 0.03,$$

leading to a deviation between experiment and theory of $1.03 \pm 0.44 = 2.34\,\sigma$.

Independently of the possibility of extra Z bosons, radiative corrections incorporating a parametrization of new physics beyond the standard Model have also been included [35] for $^{133}_{55}Cs$:

$$Q_W^{th} = -73.09 - 0.8S - 0.005T \pm 0.03. \tag{8}$$

The fact that the sensitivity to the T parameter nearly cancels is in fact accidental and is connected with the particular number of neutrons and protons present in the cesium nucleus. A contribution coming from the S parameter does not seem to be a likely candidate for interpreting the deviation $Q_W^{exp} - Q_W^{th}$, since the size of the contribution needed $S = -1.3 \pm 0.55$ is in disagreement with high energy data. Indeed the analysis of Langacker [37] leads to $S = -0.17 \pm 0.17$ if $M_Z < M_H < 150$ GeV and values very close for larger Higgs boson mass. On the other hand, this deviation could be interpreted as an indication for an extra neutral gauge boson [38], whose mass lies in the range of hundreds of GeV. Such an interpretation does not contradict high energy physics results.

7 New activity in atomic theory

The suggstion of a 0.4% theoretical uncertainty made by the Bouder group raised questions about small corrections to the prediction neglected so far. This prompted several theoretical groups to reconsider the problem.

A. Derevianko [40] was the first to evaluate the Breit correction (the magnetic interaction between all electrons) and to announce an unnegligible contribution to Q_W, $\simeq -0.6\%$ rapidly confirmed by other groups [41,42,46].

The effect of the neutron distribution, already examined [e], was revisited and confirmed to be small for cesium, -0.0018 with an uncertainty ≤ 0.001 [46], in agreement with [43,44].

Theorists of relativistic many-body calculations still agree on the value of $\mathcal{E}_1^{pv} = \frac{-N}{Q_W} E_1^{pv}$ within a 1% level of precision [42,45,46]. One group now claims an accuracy improved at the level of 0.5% [46].

New calculations now include the contribution of radiative corrections (QED type) to the electron self-energy and to the electron-Z_0 vertex [47,48,49], previously omitted. Those corrections also are unnegligible since they amount to -0.85% with about 10% accuracy.

Once the experimental result is reinterpreted at the light of the latest theoretical results Q_W^{ex} becomes:

$$Q_W^{exp} = -72.71 \pm 0.29_{exp} \pm 0.39_{theor}\ .$$

The deviation $Q_W^{exp} - Q_W^{th}$ becomes less than one σ [47,49]. At the sight of this new result, the lower limits on the mass of a possible Z' boson have been reanalyzed [39] and found comparable to those deduced from the four LEP experiments.

In spite of this apparently perfect agreement with the SM model, we would like to underline two reasons (one theoretical and the other experimental) why it may be somewhat too early to consider such an agreement as definitely well established.

1) *A slight risk of double counting in the radiative-correction evaluation.*

[e] The calculations of ref [27] allow for a difference between the proton and the neutron distributions in the Cs nucleus. They take for the protons a Fermi type distribution reproducing the rather accurate experimental data. In absence of any accurate data for the neutrons, they rely on a theoretical distribution obtained from mean-field theory which agrees with the proton data. With this slightly larger neutron radius $\mathcal{E}_1^{pv}$ is altered by -0.08%.

The calculation of the atomic factor sketched above, includes QED radiative corrections relative to Z_0 exchange between the electron and the nucleus considered like free nucleons confined inside a spherical volume. The SM test is based on the magnitude of the deviation $Q_W^{exp} - Q_W^{th}$, where Q_W^{th} itself incorporates electroweak corrections, $e-\gamma, e-W, e-Z_0$, hence also QED corrections. A global calculation incorporating all corrections to the parity violating electron-nucleus interaction would seem to be more rigorous.

2) *Necessity of independent measurements.*

- Cross-check of the ratio β/M_1^{hf} seems important. As mentioned earlier the Boulder experiment provides a determination of β/M_1^{hf} which, combined with the accurately known semi-empirical value of $M_1^{hf}(s.e.)$ leads to an absolute determination of the vector polarizability β. Recently an independent semi-empirical determination of β [51] has given a result which differs by $(0.7 \pm 0.4)\%$. Though small, such a difference, considered alone, is sufficient to reduce the deviation $Q_W^{exp} - Q_W^{th}$ from 2.2 to 0.9 σ [46]. This is enough to emphasize how important would be a new empirical determination of β/M_1^{hf}, if possible in presence of a much reduced background.

- An independent determination of Q_W is crucial. We cannot understate how welcome would be a new determination of $Q_W(Cs)$ using a totally different method. Our current experiment developed in Paris should actually fulfill this objective.

8 New PV manifestation in cesium using stimulated-emission detection

A new experiment currently performed on the Cs $6S \to 7S$ transition, in our group at Paris, differs very significantly from both our first 82-83 experiment and the Boulder experiment. While all atomic parity violation experiments on highly forbidden transitions in a Stark field have used so far the detection of fluorescence signals, this experiment exploits the possibilities of the Λ-type three-level system $6S_{1/2} - 7S_{1/2} - 6P_{3/2}$ in interaction with two lasers. One intense laser with linear polarization $\hat{\epsilon}_{ex}$ excites the forbidden transition in a longitudinal $\mathbf{E}$ field and the second laser probes the angular anisotropy induced in the excited state. The probe laser is amplified and its polarization $\hat{\epsilon}_{pr}$ altered. A precise analysis of the polarization modification is performed on the transmitted beam. In order to provide suitable conditions for PV measurements, a large gain for the probe has to be achieved, hence the realization of this experiment with a pulsed excitation laser and a gated probe. The right-left asymmetry is a rotation of the linear polarization of the probe $\hat{\epsilon}_{ex}$, which can be detected at each laser pulse with a dual channel polarimeter operating in balanced mode (see Fig. 5). The effect is the manifestation of the pseudoscalar $(\hat{\epsilon}_{ex} \cdot \hat{\epsilon}_{pr})(\hat{\epsilon}_{ex} \wedge \hat{\epsilon}_{pr} \cdot \mathbf{E})$ present in the gain, which is responsible for contributions of opposite signs to the amplified intensity measured in both channels.

This effect can be understood on the basis of simple symmetry considerations. The excitation polarization $\hat{\epsilon}_{ex}$ and the $\mathbf{E}$ field determine two symmetry planes for the experiment. Without parity violation one would expect the excited Cs vapor to have its optical axes contained in those planes. If this were the case, a probe beam linearly polarized with $\hat{\epsilon}_{pr} \parallel \hat{\epsilon}_{ex}$ would pass through the vapor without alteration of its polarization. Actually, due to parity violation acting during the excitation process, the optical axes of the excited vapor are tilted with respect to the symmetry planes and it is this tiny tilt angle $\theta^{pv} = Im\ E_1^{pv}/\beta\mathbf{E}$, odd under $\mathbf{E}$-reversal, which has to be determined. As a consequence of this tilt, while the probe beam passes through the vapor its polarization rotates towards the axis of larger gain. This causes an imbalance at the output of the polarimeter, odd in $\mathbf{E}$ reversal, which is measured. It is precisely calibrated by measuring the imbalance induced by a small, precisely known angle, in identical conditions. An attractive feature of this experiment is that the right-left asymmetry itself, A_{LR},

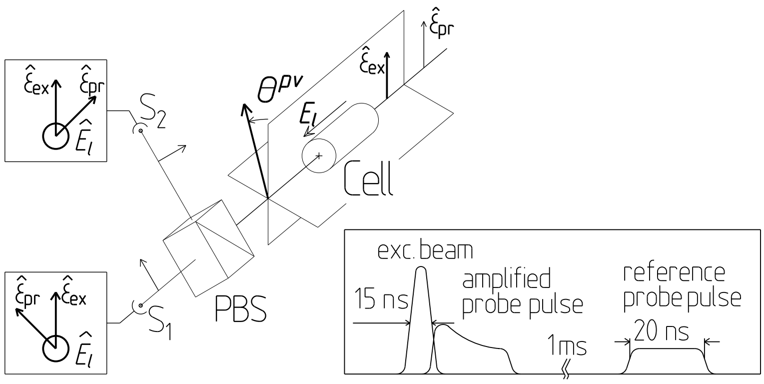

Figure 5: Schematic of the experiment showing the two orthogonal symmetry planes defined by the electric field **E** and the linear excitation polarization $\hat{\epsilon}_{ex}$. APV gives rise to a tilt θ^{pv} of the optical axes of the excited vapor out of those planes. The incoming probe polarization $\hat{\epsilon}_{pr}$ provides a superposition of the right and left-handed $\hat{\epsilon}_{ex}, \hat{\epsilon}_{pr}, \mathbf{E}$ configurations analyzed. The probe amplification difference is directly extracted from the optical signals S_1, S_2, recorded in each channel of the Polarizing Beam Splitter (PBS). Inset: timing of the experiment repeated at $\sim$ 100 Hz. (Figure adapted from Guéna *et al.* [53]).

is amplified while the probe propagates through the optically thick vapor. Therefore, instead of being a decreasing function of the applied field as in usual fluorescence experiments, A_{LR} is transformed by stimulated-emission detection into an increasing function of **E** [54].

The present result for $Im\ E_1^{pv}/\beta$, now reaching 8.4% statistical accuracy, agrees with the more accurate Boulder result. Checks for systematic effects are described in ref.[53]. We believe that this experiment has not yet reached its ultimate precision, several improvements are currently being implemented in order to obtain a better S/N ratio and the repetition rate will be increased. The one percent Q_W-precision objective looks within reach.

This experiment will provide a measurement totally independent from that recently performed on the cesium atom at Boulder, and thereby an important cross-check. A conceptual difference is that the detected observable is directly the asymmetry itself as opposed to a modulation with reversals of order 6×10^{-6} in the total transition rate. Moreover the present approach avoids the difficulty met there regarding M_1 systematics and line-shape dependent effects.

9 Experiments in progress and new proposals

9.1 Work in progress on a chain of rare earth isotopes

It has not been possible, yet, to test an important pediction of the SM model concerning the variation of Q_W along a string of isotopes. It has been suggested [55] that in rare earth spectra one can find atomic states of opposite parity which are either degenerate or incompletely degenerate. The first situation occurs in Dy, Z=66, and the latter in Yb, Z=70. It is expected thats this degeneracy enhances the PV effect, thus making possible precise measurements along a chain of isotopes. Both elements, Dy and Yb, have chains of seven stable isotopes. In first approximation, the ratios of the E_1^{pv} matrix elements should provide the ratio of the weak charges, without invoking atomic physics calculations, made complex in the present case by configuration mixings. Furthermore, while the nuclear anapole moment has been detected up to now only in ^{133}Cs, an even neutron-number isotope, this should provide some means of observing it in a new element and in odd-neutron number isotopes.

The search for parity violation in atomic dysprosium is conducted by the Berkeley group by observing time-resolved quantum beats between the nearly degenerate opposite parity states [56] and by searching for the contribution arising from an E_1^{pv}-Stark interference having a characteristic signature. The search in ytterbium, also conducted in Berkeley [57], presents analogies with that performed in cesium. The spectrum of this atom is much simpler than that of other rare earth atoms, so that the theoretical predictions should be more reliable. The transition chosen is the $6s^2\ ^1S_0 \rightarrow 5d6s\ ^3D_1$ highly forbidden transition because of the existence of an odd-parity state nearly degenerate with the upper state. This coincidence is expected to lead to to a significant enhancement of the parity mixing amplitude and the effect should be reinforced by the existence of a strong E_1 amplitude between the mixed state and the ground state[58]. The E_1^{pv} amplitude is predicted to be 100 times larger than in Cs. Important exploratory work has already been carried out and the magnetic dipole amplitude of this highly forbidden transition has already been measured[59].

9.2 AC Stark effect in a single Ba$^+$ ion

A totally different approach has been undertaken in Seattle[60]. It makes use of the remarkable properties of single trapped ions. In this experiment, a single Ba$^+$ ion is confined in a radiofrequency trap and laser cooled so that its residual motion is localised to a small fraction of an optical wavelength. Parity violation should manifest itself via a a frequency shift in the ground state Zeeman splitting. This ac Stark shift is induced by two independent standing wave laser fields tuned very close to the $6S_{1/2} \rightarrow 5D_{3/2}$, E_2 transition. Both beams derived from the same laser are incident from perpendicular directions. The spatial phase from the stronger laser field $E^{'}$ is adjusted so as to provide an antinode at the location of the ion. Having no gradient $E^{'}$ selectively drives the quadrupolar electric transition. The second field, with a smaller amplitude $E^{''}$, is chosen to create a node in the same place. The associated gradient drives the quadrupolar electric transition. It is the interference between the two corresponding amplitudes which gives rise to the PV "light shift" of the Larmor frequency in the ground state. Reversal properties of this shift can reveal the pseudoscalar characteristics of its weak interaction origin.

In this experiment, it is expected that the considerable loss accepted on the atomic density as compared to traditional situations, will be compensated by the greatly increased coherence time of the transition (the lifetime of the $5D_{3/2}$ state, $\tau = 50$ s) and by the possibility to focus tightly the laser field at the location of the laser cooled ion. This exciting, but also very demanding project is still at an exploratory stage[61].

9.3 Prospects with cooled and trapped atoms

Cooling and trapping techniques provide the single possibility of confining a sample of radioactive atoms in free space and the possibility of making measurements with radioactive Cs isotopes or even francium atoms. With Z=87, francium is expected to lead, due to the fast increase with the atomic number[3], to PV effects 18 times larger than cesium. In addition, since atomic structure calculations for alkali are (excluding H and He) the most precide available, we can reasonably expect a theoretical prediction of its weak charge as precise as that for cesium. This added to the fact that many isotopes can be produced, makes this element often considered as one of the most promising candidates for forthcoming experiments.

Fr atoms, either obtained from a radioactive source or produced on line by an accelerated ion beam colliding a target, are produced at a limited rate with a thermal or superthermal velocity distribution. The first prerequisite is to avoid their spreading out in space and their loss inside the wall. Successive attempts to load Fr atoms in a neutral atom trap have already made possible the observation of several Fr allowed transitions, and have thus led to precise spectroscopic measurements[62,63]. To our opinion, the observation of the forbidden $6S \rightarrow 7S$ line with a sample of cold cesium atoms will represent an important preliminary step to assess the feasibility of a PV measurement in the Fr $7S \rightarrow 8S$ forbidden transition. There has been definite proposals suggesting to extract the cold atoms from the trap under the form of a slow, cold atomic beam, and to make it pass through the interaction region where the PV experiment takes place between two capacitor plates[64].

9.4 Static manifestations of the electroweak interaction

When an atom is placed in a chiral environment, Sandars' theorem [65] no longer holds and Parity Violation can manifest itself by an energy shift of its atomic levels. Although shifts of this kind still require considerable experimental efforts to be detected, we mention their existence because of their conceptual interest.

For chiral molecules one expects a small energy shift between the two mirror-image enantiomers. It has been searched for by comparing vibrational frequencies of the right and left-handed species of CHFClBr molecules [66]. Experiments in more favorable conditions are in progress.

When Cs atoms are trapped in a solid matrix of ^{4}He of hexagonal symmetry, two applied **E** and **B**, static fields and the crystal axis $\hat{n}$ create a chiral environment around each atom. In these conditions a linear Stark shift proportional to the Cs nuclear anapole moment and to the (T-even) P-odd pseudoscalar $(\hat{n} \cdot \mathbf{B})(\hat{n} \cdot \mathbf{B} \wedge \mathbf{E})/B^2$ has been predicted [67].

Both effects could provide a static manifestation of the electroweak interaction, which is still missing.

10 Conclusion

In this paper we have summarized the continuous progress of APV in cesium since its very beginning. This illustrates how important has been the interplay between experiment and theory for finally achieving a precise determination of the cesium weak charge. Clearly Atomic Parity Violation has now a significant role to play in precision tests of the Standard Model and exploration of possible alternatives. In particular, the exchange of new Z's can be valuably constrained by such a result. If cesium plays at present a major role, it is because of its simple atomic structure, and the existence of many precise spectroscopic data leading to numerous tests of the atomic calculations. New measurements and calculations are expected: the recent 0.6% accurate result needs to be confirmed. A project is in progress at ENS-Paris [3,52]; since it is based on very different principles the systematics have no reason to be the same. Furthermore, very interesting projects in other elements, for instance Ba^+ [60], rare earths [56,57], francium and radioactive isotopes of Cs are also in progress or under serious consideration.

In addition, one must bear in mind that APV has provided the first determination of the cesium nuclear anapole moment, a *static* parity violating quantity which describes the chirality of the nuclear magnetization induced by the parity violating nuclear forces.

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